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On Antiplane Deformations of an Elastic Material with Rigid Fibers Considering Surface Energy and Non-Perfect Contact

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Abstract

Within the linear Gurtin–Murdoch model of surface elasticity we consider the antiplane deformations in an elastic matrix with an rigid cylindrical fiber subjected by external force. Influence of an surface elastic moduli on the stress distribution and the adhesion force is analyzed.

1 Introduction

The surface elasticity model proposed by [Gurtin and Murdoch(1975)] found various applications in modelling of material behavior at the nanoscale, see [Duan et al.(2008)Duan, Wang, and Karihaloo, Wang et al.(2011)Wang, Huang, Duan, Yu, Fe Javili et al.(2012)Javili, McBride, and Steinmann, Eremeyev(2016)], as in this case the ratio of a number of atoms near the surface/interface to a number of atoms in volume may be large. In other words, as at the nanoscale the surface related phenomena play a crucial role one has to modify properly the material model. It is worth to note that the Gurtin–Murdoch model was generalized in many directions, let us note here the model by [Steigmann and Ogden(1997), Steigmann and Ogden(1999)]. Unlike the Gurtin–Murdoch model the latter model takes into account the bending resistance of the surface or interface. So from mechanical point of view the Gurtin–Murdoch model corresponds to an elastic membranes attached to the body surface whereas the Steigmann–Ogden model is similar to a shell attached to the body boundary. Both models found many applications in description of behavior of materials with cracks, nanosized inclusions and holes, and of nanocomposites.

The presence of surface stresses may significantly influence on the material properties at the nanoscale, see, e.g., [Kushch et al.(2014)Kushch, Chernobai, and Mishuris, Chatzigeorgiou et al.(2015)Chatzigeorgiou, Javili, and Steinmann, Han et al.(2018)Han, Mogilevskaya, and Schillinger, Huang and Wang(2012)] for nanocomposites, as well it changes the behaviour of solutions of various boundary-values problems. Let us only mention here the papers by [Kim et al.(2010b)Kim, Schiavone, and Ru, Kim et al.(2010a)Kim, Schiavone, and Ru, Kim et al.(2011)Kim, Schiavone, and Ru], where was shown that unlike classic linear elasticity the stresses remain finite at the mode III crack tip.

In this note we consider anti-plane deformations of an elastic material with an embedded fiber considering the surface elasticity within the Gurtin–Murdoch problem and non-perfect contact. In order to model the non-perfect contact between elastic matrix and rigid fiber we introduce the adhesion energy as a function of relative displacement, see [Wei and Hutchinson(1998)]. Let us note that imperfect interfaces (nonclasic) transmission conditions) are appeared in various situation with the high contrast properties and can be evaluated phenomenological from the first principles or atomistic modeling, see, e.g., [Sfyris et al.(2014)Sfyris, Sfyris, and Galiotis, Arroyo and Belytschko(2002)] or by asymptotic methods developed by [Mishuris(2004), Movchan and Movchan(1995), Sonato et al.(2015)Sonato, Piccolroaz, Miszuris, and Mishuris]. Such transmission conditions may significantly change the asymptotic behaviour of the problem solutions near the singular points (crack tips, defects, specific singular points of the domain boundary, etc.), see [Mishuris(1997), Mishuris(1999), Mishuris(2001), Mishuris and Kuhn(2001), Mishuris et al.(2006)Mishuris, Movchan, and Movchan]. Here we consider one of the simplest transmission conditions to demonstrate the influence of the surface elasticity. For axisymmetric problem the solution was obtained in an analytical form. Here the tangent shear stresses at the interface between matrix and fiber are constant. For a non-axisymmetric problem it is not the case. Some solutions suggest that such models may be sensitive to geometrical imperfections of the cross-section shape which may lead even to appearance of singular surface stresses.

2 Governing equations

Let an elastic solid occupy volume V in R^3 with the boundary $A = \partial V$ and let R^3 designate the three-dimensional space. We assume that the solid may consist of two parts, V^+ and V^- separated by a smooth interface I. We attribute to the interface surface strain energy density. In addition, we also assume the presence of surface energy and surface stresses on the part of boundary, that is on $A_s \subset A$. We consider infinitesimal deformations of the solid described by the displacement field

$$\mathbf{u} = \mathbf{u}(\mathbf{x}), \tag{1}$$

where $\mathbf{u}$ is a twice differentiable vector-function of displacements, $\mathbf{x}$ is the position vector.

In what follows, we use the classic constitutive equations of a linear elastic isotropic body in the bulk

$$\mathcal{W} = \mu \mathbf{e} : \mathbf{e} + \frac{1}{2}\lambda(\operatorname{tr}\mathbf{e})^2, \quad \boldsymbol{\sigma} \equiv \frac{\partial \mathcal{W}}{\partial \mathbf{e}} = 2\mu\mathbf{e} + \lambda\,\mathbf{I}\operatorname{tr}\mathbf{e}, \quad \mathbf{e} = \frac{1}{2}\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right), \tag{2}$$

where $\mathcal{W}$ is the strain energy density, $\boldsymbol{\sigma}$ is the stress tensor, $\mathbf{e}$ is the strain tensor, $\mathbf{I}$ is the unit second-order tensor, the double dot stands for scalar (inner) product of two second-order tensors, λ and μ are Lamé moduli, $\mu > 0$, $3\lambda + 2\mu > 0$, ∇ is the 3D nabla operator, and tr is the trace operator.

In order to model the interface between two solids we use the model based on the surface elasticity model by [Gurtin and Murdoch(1975)] considering the nonlinear traction-separation law. According to linearized model of surface elasticity by [Gurtin and Murdoch(1975)], the surface strain energy density $\mathcal{W}_s$ and surface stress tensor $\boldsymbol{\tau}$ are defined as follows, see also [Duan et al.(2008)Duan, Wang, and Karihaloo],

$$\mathcal{W}_s = \mu_s \boldsymbol{\epsilon} : \boldsymbol{\epsilon} + \frac{1}{2}\lambda_s(\operatorname{tr}\boldsymbol{\epsilon})^2, \quad \boldsymbol{\tau} \equiv \frac{\partial \mathcal{W}_s}{\partial \boldsymbol{\epsilon}} = \mu_s\boldsymbol{\epsilon} + \lambda_s\mathbf{A}\operatorname{tr}\boldsymbol{\epsilon}, \quad \boldsymbol{\epsilon} = \frac{1}{2}\left((\nabla_s\mathbf{u})\cdot\mathbf{A} + \mathbf{A}\cdot(\nabla_s\mathbf{u})^T\right), \tag{3}$$

where λ_s and μ_s are the surface elastic moduli called also surface Lamé moduli, ∇_s is the surface nabla operator, $\mathbf{A} \equiv \mathbf{I} - \mathbf{n} \otimes \mathbf{n}$ are the surface unit second-order tensor, $\mathbf{n}$ is the unit vector of outer normal to A_s, the symbol $\otimes$ designates the tensorial product between two vectors and $\boldsymbol{\epsilon}$ is the infinitesimal deformations associated with the surface.

For modelling of a non-perfect interface that is the case when discontinuities in displacements across the interface may exist, we propose the following nonlinear models. We attribute to the non-perfect interface two displacement fields that is one-sided limits $\mathbf{u}^- = \lim \mathbf{u}(\mathbf{x}^-, t)_{\mathbf{x}^- \to I}$ and $\mathbf{u}^+ = \lim \mathbf{u}(\mathbf{x}^+, t)_{\mathbf{x}^+ \to I}$, where $\mathbf{x}^\pm \in V^\pm$. The surface strain energy density is assumed to be

$$\mathcal{W}_i = \mathcal{W}_s^- + \mathcal{W}_s^+ + K(d), \quad d = |[\![\mathbf{u}]\!]|,$$

where $[\![\mathbf{u}]\!] = \mathbf{u}^- - \mathbf{u}^+$ and K is a nonlinear function describing the adhesion similar to [Wei and Hutchinson(1998)]. Moreover, note that

$$K(d) = K_0 + O(d^2) \quad d \to 0. \tag{4}$$

The adhesion force is given by

$$f = f(d) \equiv \frac{\mathrm{d}K}{\mathrm{d}d}.$$

The typical shape of the traction-separation law that is shown in Fig. 1, see, e.g., [Nase et al.(2016)Nase, Rennert, Naumenko, and

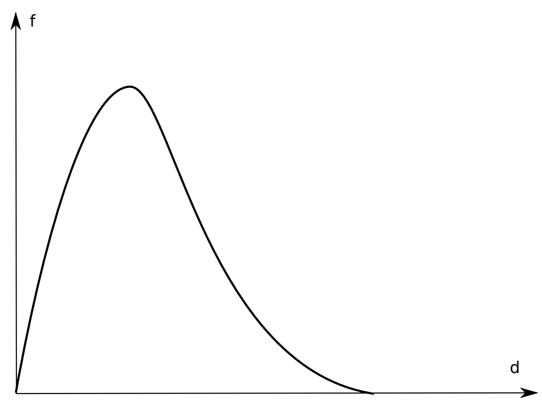

Figure 1: Classic traction-separation law.

The interfacial boundary conditions follow from the variational principle

$$\delta \left[\int_V \mathcal{W} \mathrm{d}V + \int_I \mathcal{W}_i \mathrm{d}a \right] = 0.$$

At the interface we have

$$\mathbf{n} \cdot \boldsymbol{\sigma}^+ = \nabla_s \cdot \boldsymbol{\tau}^+ - f(d) \frac{1}{d}(\mathbf{u}^+ - \mathbf{u}^-), \quad \mathbf{n} \cdot \boldsymbol{\sigma}^- = -\nabla_s \cdot \boldsymbol{\tau}^+ - f(d) \frac{1}{d}(\mathbf{u}^+ - \mathbf{u}^-). \tag{5}$$

Note that $f(d)/d$ exhibits the linear behaviour (compare to (4)). More complex interfacial energy and corresponding natural boundary conditions are also discussed in the literature, see, e.g., [Lurie et al.(2009)Lurie, Volkov-Bogorodsky, Zubov, and Eremeyev et al.(2016)Eremeyev, Rosi, and Naili, Placidi et al.(2014)Placidi, Rosi, Giorgio, and Madeo, Eremeyev(2016)] and the reference therein.

3 Antiplane deformation

Let us consider anti-plane deformations as it highlights the main features of the model and simultaneously is simple enough for analysis. Here and in what follows x_1, x_2 and x_3 are the Cartesian coordinates and $\mathbf{i}_k$ are corresponding unit base vectors, $\mathbf{n} = \mathbf{i}_1$. For an anti-plane motion, the vector of displacement takes the form as in [Achenbach(1973)]

$$\mathbf{u} = u(x_1, x_2)\mathbf{i}_3. \tag{6}$$

From (6), it follows that

$$\nabla \mathbf{u} = u_{,\alpha}\mathbf{i}_\alpha \otimes \mathbf{i}_3 = \nabla u \otimes \mathbf{i}_3, \quad \nabla \cdot \mathbf{u} = 0, \quad \mathbf{e} = \frac{1}{2}(\nabla u \otimes \mathbf{i}_3 + \mathbf{i}_3 \otimes \nabla u), \quad \boldsymbol{\epsilon} = \frac{1}{2}(\nabla_s u \otimes \mathbf{i}_3 + \mathbf{i}_3 \otimes \nabla u_s).$$

Hereinafter we used the notation $u_{,\alpha} = \frac{\partial u}{\partial x_\alpha}$, and Greek indices take values $1, 2$. For anti-plane deformation $\nabla_s = \boldsymbol{\tau}\partial/\partial s$, where s is the arc-length parameter and $\boldsymbol{\tau}$ is the unit tangent vector to the cross-section contour. For the anti-plane shear deformation (6), the equilibrium equations reduce to one scalar equation

$$\mu \Delta u = 0, \tag{7}$$

where $\Delta u = u_{,11} + u_{,22}$ is the 2D Laplacian.

3.1 Axisymmetric problem

Such anti-plane state may be realized considering for deformations of an elastic circular cylinder with a rigid fiber extracted from the medium and subjected by a force F, see Fig. 2. It is clear that for a fiber with circular cross-section the deformation is axisymmetric that is

$$u = u(r),$$

and $W_s = 0$ since $\varepsilon = 0$. As a result, the external force F depends on the relative displacement exactly following the law $F = f(d)$, $d = u_2 - u(r_1)$, where $u_2 > 0$ displacement of the fiber.

For such a problem we have that

$$\nabla_s = \frac{1}{r}\mathbf{e}_\phi \frac{\partial}{\partial \phi}, \quad \boldsymbol{\tau}_+ = \mathbf{0}, \quad \boldsymbol{\tau}_- = \mathbf{0}.$$

In addition, we assume that $\mathbf{u}_-$ represents to the rigid motion of the extracted fiber, and thus $\mathbf{u}_- =$ const. Hereinafter, r, ϕ and $\mathbf{e}_r$, $\mathbf{e}_\phi$ are the polar coordinates and the corresponding unit base vectors, respectively. As

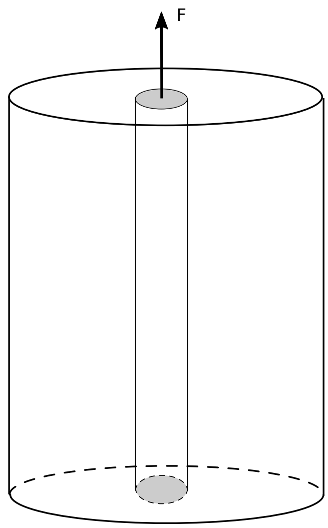

Figure 2: A fiber with circular cross-section in a matrix.

a result, the only remaining boundary condition takes the form

$$\mu u^+_{,r} = -f(d).$$

Obviously, the axisymmetric problem does not contain the surface elastic modulus μ_s. In other words, this means that the linear Gurtin–Murdoch elasticity model does not "feel" such a symmetric interface.

The solution of the problem can be sought in the form as

$$u(r) = C_1 \ln \frac{r}{R}, \quad u(R) = 0,$$

where C_1 should found from the equation

$$\mu C_1 \frac{1}{r_1} = -f(d).$$

Let us note that $u(r_1) \leq u_2$ and $C_1 \leq 0$.

As a results we get

$$d = u_2 - C_1 \ln \frac{r_1}{R}.$$

Thus,

$$C_1 = (u_2 - d)/\ln \frac{r_1}{R}$$

and the basic equation takes the form

$$\mu \frac{u_2 - d}{r_1 \ln \frac{R}{r_1}} = f(d) \tag{8}$$

or in more simple form

$$u_2 - d = \bar{f}(d),$$

where $\bar{f}(d)$ is re-scaled $f(d)$ in accordance with (8). The possible solutions are shown in next figure. For small u_2 there is only one solution of this equation. The same conclusion can be made for large enough values of u_2.

On the contrary, there exist two solutions of (8) for $u_2^* \leq u_2 < u_2^{**}$, see Fig. 3. Here u_2^* and $< u_2^{**}$ depends on the both mechanical and geometrical parameters of the problem and the adhesion law.

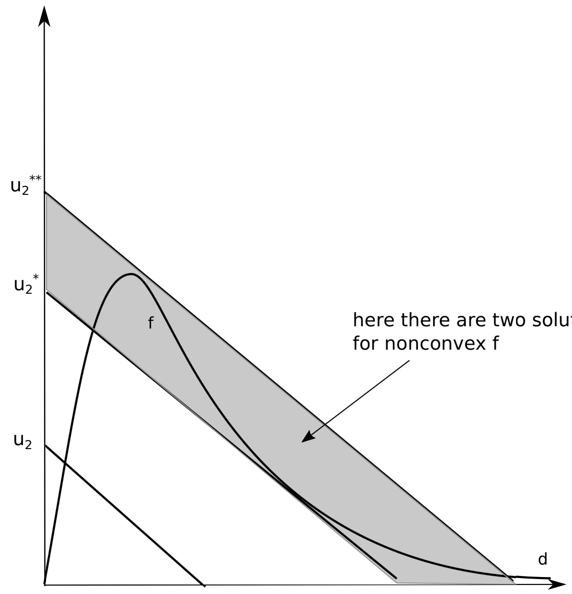

Figure 3: Traction-separation law: grey area denotes the case of two solutions.

3.2 Non-axisymmetric problems

If the boundary is not circle, then the displacement depends on two variables

$$u = u(r, \phi),$$

and $\boldsymbol{\tau}_+ \neq 0$, in general. The boundary condition take the form

$$\mu u_{,n}^+ = \mu_s u_{,ss}^+ - f(d),$$

where $u_{,n}$ and $u_{,s}$ are normal and tangent derivatives, respectively. Obviously, here the solution depends on μ_s and can be obtained by similar methods developed by [Andreeva and Miszuris(2018), Andreeva and Miszuris(2017)]. Note that the result may be quite different with that presented here for axisymmetric mode III problem.

4 Conclusions

We have considered anti-plane deformations of an elastic matrix with embedded rigid fiber taking into account surface stresses within the Gurtin–Murdoch surface elasticity and adhesion energy. The presented analysis shown that

- even for small deformations there is a possibility of multiple solutions due to nonlinearity in adhesion force;

- for circular cross section and axisymmetric problem the value of the surface shear modulus has no influence on the solution, whereas for any geometrical imperfections violating the axial symmetry it changes the solution behavior. In other words, this means the necessity of consideration of surface elasticity models for non-perfect inclusions.

Note that the same two conclusions can be made not only for mode III deformations of such a geometry but for arbitrary axisymmetric problems, as follows from (5).

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